

# GEOMETRICAL OPTICS

## CH-1

### The nature and propagation of light

Light: is that aspect of radiant energy of which a human observer is aware through the visual sensations which arise from stimulation of the eye.

wave front: is defined as the locus of points ,all of which are lie in the same phase.

Huygen's principle: Huygens proposed the rule that each point on a wave front may be represented as a new source of waves.

The velocity of light: the velocity of propagation of light in free space is one of the fundamental constants in nature is about  $3 \times 10^8 \text{m/s}$ .

Index of refraction: the ratio between the velocity of light in vacuum to the velocity of light in the medium.  $n = \frac{c}{v}$  ,n-index of refraction,

c-velocity of light in space, v-velocity of light in the medium.

wavelength of the light  $\lambda = \frac{c}{f}$

$$1\mu\text{m} = 10^{-6}\text{m}$$

$$1\text{nm} = 10^{-9}\text{m}$$

$$1\text{A}^0 = 10^{-10}\text{m}$$

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### The electromagnetic spectrum

The range of wavelengths of the electromagnetic waves extends from the waves which have longest wavelength (radio waves), to the waves which have shortest wavelength (Gamma waves).

types of electromagnetic waves:

1-**Radio waves**: These waves were used by Hertz which extends from a few Hz up to about  $10^9$  Hz ( $\lambda$  from many kilometers to 0.3 m or so), these are generally emitted by using oscillating electrical circuits, and used in communication systems, TV, radio.

2-**Microwaves**: The microwave region extends from about  $10^9$  Hz up to about  $3 \times 10^{11}$  Hz, the corresponding wavelength goes from roughly 30 cm to 1 mm. radiation capable of penetrating the earth's atmosphere. microwaves are therefore of interest in space-vehicle communications as well as radio astronomy.

3-**Infrared waves**: the infrared region extends roughly from  $3 \times 10^{11}$  Hz up to  $4 \times 10^{14}$  Hz, this region is often subdivided to four regions: a-near infrared: i.e near the visible (780-3000) nm.

b-intermediate IR: (3000-6000) nm. c-far IR: (6000-15000) nm.

d-extreme IR: (15000 nm-1 mm).

radiant energy at the wavelength extreme can be generated using microwave oscillator in incandescent sources.

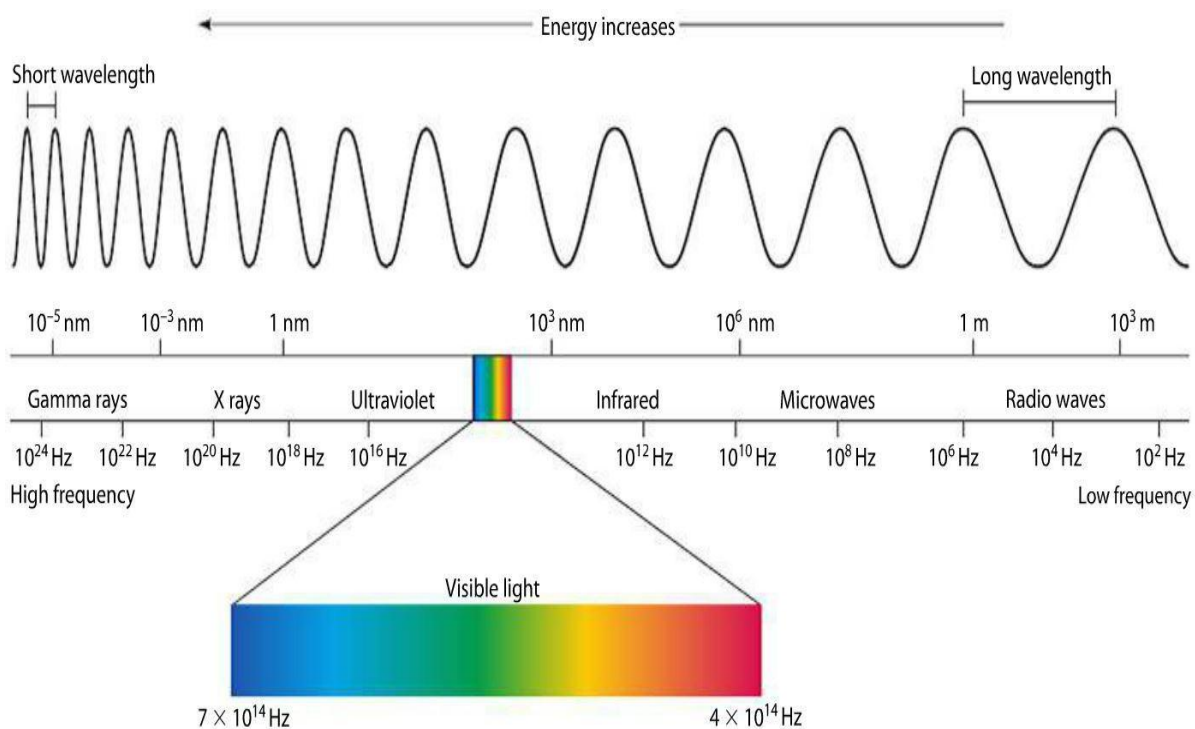
4-**Visible waves**: visible light corresponds to electromagnetic radiation in the narrow bands of frequencies from about  $3.84 \times 10^{14}$  Hz to  $7.69 \times 10^{14}$  Hz. it is generally produced by rearrangement of the outer electrons in atoms and molecules. the human eye can sense this region of the spectrum.

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5-**Ultraviolet waves**: this region extends from ( $8 \times 10^{14}$  Hz to about  $3 \times 10^{17}$  Hz). ultraviolet or UV rays from the sun will thus has more than enough energy to ionize atoms in the upper atmosphere, and in so doing create ionosphere. ultraviolet is detectable via fluorescent screens, photographic emissions and photocells.

6-**X-rays**: extends in the range frequency from  $3 \times 10^{17}$  Hz, x-rays photons are emitted by an atom or molecule when the inner, tightly bound electrons undergo transitions. these are the highest energy ( $10^4$  eV to about the lowest wavelength electromagnetic radiations  $10^{19}$  eV).

7-**Gamma rays**: they are electromagnetic waves of nuclear origin, and emitted by nuclei of the radiant particles, for example,  $^{137}\text{Cs}$ ,  $^{60}\text{Co}$  during special nuclear interactions.



## **GEOMETRICAL OPTICS**

### **The electromagnetic spectrum**

The nature and propagation of light until the time( Isshac Newton)(1727)most scientists thought that the light consisted of a stream of particles emitted by light sources and traveled outward from the source in a straight lines .by the middle of 17th century (christian Huygens) showed that the laws of reflection and refraction could be explained on the basis of wave theory .the wave demonstrated the phenomenon of interference and diffraction of light. in 1873Maxwell explained that the oscillating electrical circuit showed radiate electromagnetic waves. the velocity of propagation of the waves could be computed from purely electrical and magnetic measurement and it appeared close to  $3 \times 10^8$ m/s. the classical electromagnetic theory failed to account for the phenomenon of photoelectric emission that is ejection of electrons from a conductor by light incident on it's surface.

in 1905 Enistein extended an idea proposed five years earlier by plank and postulated that the energy in the light beam.

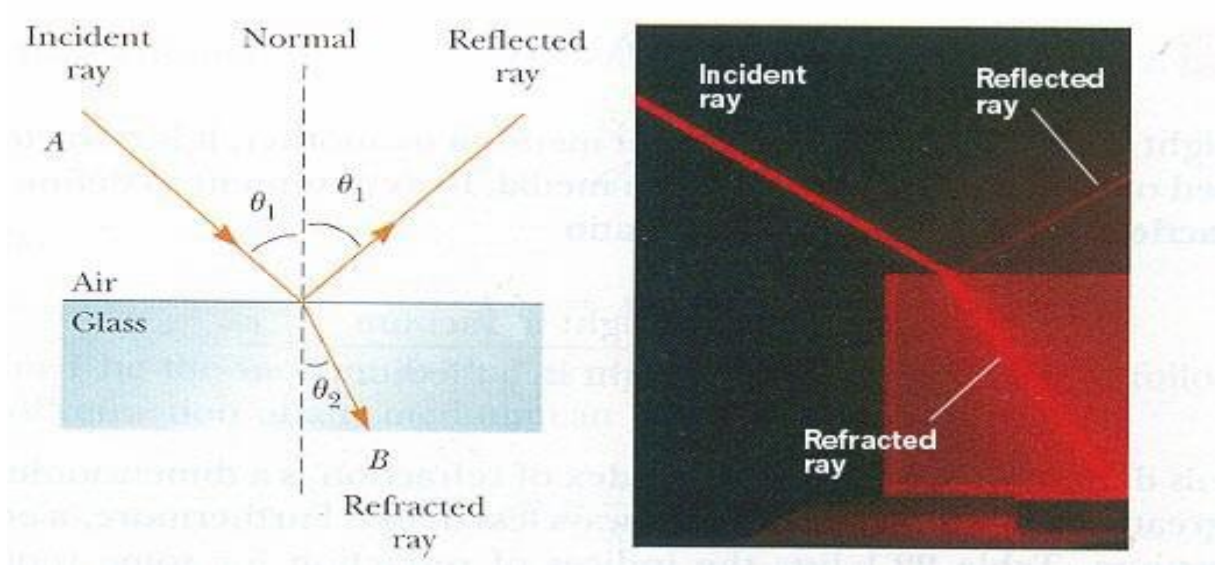
## **CH-2**

### **Reflection and refraction**

#### **Reflection and refraction at plane surface:**

The ray of light is incident on the boundary separating two different media, part of the ray is reflected and the remainder is refracted

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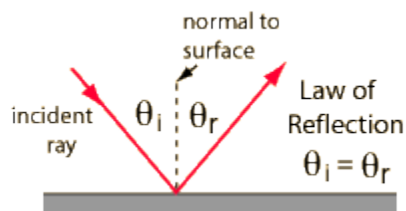
$\theta_1$ -angle of incidence,

$\theta_2$ -angle of refraction

### Laws of reflection and refraction:

1-the incident, reflected and refracted ray and the normal to the surface are all lie in the same plane.

2-the angle of reflection ( $\theta_r$ ) is equal to the angle of incidence.



3-the ratio between the sine of the angle of incidence and the sine of the angle of refraction is a constant.

$$\frac{\sin\theta_1}{\sin\theta_2} = \text{constant}$$

$$\frac{n}{n} = \text{constant}$$

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$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad \text{snell's law}$$

Ex: Light travels from air into an optical fiber with an index of refraction of 1.44. (a) In which direction does the light bend? (b) If the angle of incidence on the end of the fiber is  $22^\circ$ , what is the angle of refraction inside the fiber? (c) Sketch the path of light as it changes media

(a) Since the light is traveling from a rarer region (lower  $n$ ) to a denser region (higher  $n$ ), it will bend **toward the normal**.

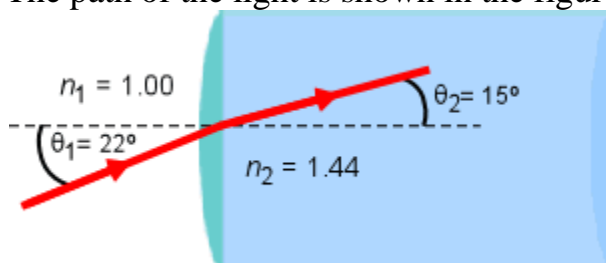
(b) We will identify air as medium 1 and the fiber as medium 2. Thus,  $n_1 = 1.00$ ,  $n_2 = 1.44$ , and  $\theta_1 = 22^\circ$ . Snell's Law then becomes

$$(1.00) \sin 22^\circ = 1.44 \sin \theta_2.$$

$$\sin \theta_2 = (1.00/1.44) \sin 22^\circ = 0.260$$

$$\theta_2 = \sin^{-1}(0.260) = 15^\circ.$$

(c) The path of the light is shown in the figure below.



EX: Light traveling through an optical fiber ( $n=1.44$ ) reaches the end of the fiber and exits into air. (a) If the angle of incidence on the end of the fiber is  $30^\circ$ , what is the angle of refraction outside the fiber? (b) How would your answer be different if the angle of incidence were  $50^\circ$ ?

### Solution:

(a) Since the light is now traveling from the fiber into air, we will call the fiber material 1 and air material 2. Thus,  $n_1 = 1.44$ ,  $n_2 = 1.00$ , and  $\theta_1 = 30^\circ$ . Snell's Law then becomes

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$$(1.44) \sin 30^\circ = 1.00 \sin \theta_2.$$
$$\sin \theta_2 = (1.44/1.00) \sin 30^\circ = 1.44 (0.500) = 0.720$$
$$\theta_2 = \sin^{-1} (0.720) = 46^\circ.$$

Notice that this time, the angle of refraction is larger than the angle of incidence. The light is bending away from the normal as it enters a rarer material.

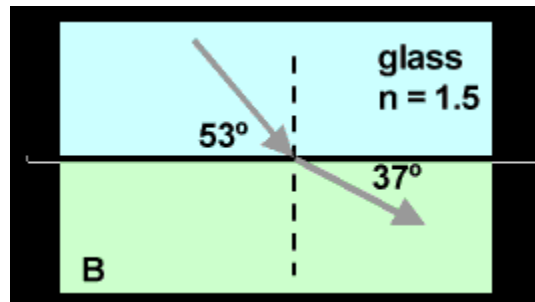
(b) Replacing the angle of incidence with  $50^\circ$  gives

$$\sin \theta_2 = (1.44/1.00) \sin 50^\circ = 1.44 (0.766) = 1.103$$

This equality cannot be met, so **light cannot exit the fiber** under these conditions

EX:

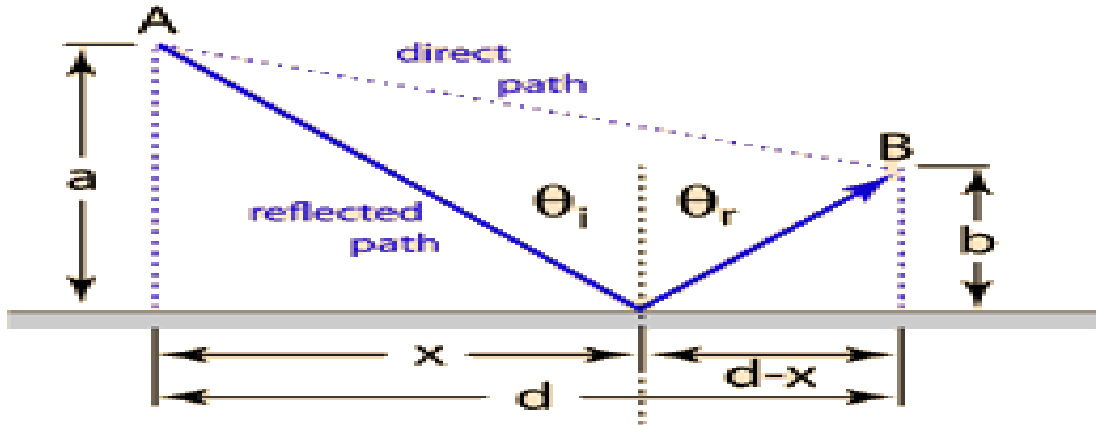
Calculate the *index of refraction* for medium B.



**Fermat's principle**: states that light travels between two points along the path that requires the least time, as compared to other nearby path.

**Derivation of the laws of reflection law according to Fermat's law:**

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Consider the light ray as shown in figure, a ray of light starting from point A .we calculate the length of each path and divide the by the speed of light to determine the time required to travel between two points.

The path length from A to B is

$$t = \frac{L}{c} = \frac{\sqrt{x^2 + a^2}}{c} + \frac{\sqrt{(d-x)^2 + b^2}}{c}$$

to minimize the time we set the derivative of the time with respect to x equal to zero (Fermat's principle).

$$\frac{dt}{dx} = 0 = \frac{x}{c\sqrt{x^2 + a^2}} + \frac{-(d-x)}{c\sqrt{(d-x)^2 + b^2}}$$

$$\frac{x}{\sqrt{x^2 + a^2}} = \frac{(d-x)}{\sqrt{(d-x)^2 + b^2}}$$

$$\sin\theta_1 = \sin\theta_2, \text{ when } \theta_1, \theta_2 \text{ are very small}$$

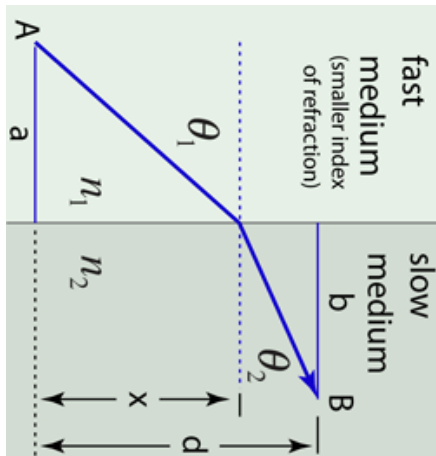
$$\theta_1 = \theta_2$$

**Derivation of snell's law according to Fermat's law:**



## GEOMETRICAL OPTICS

Fermat's Principle: Light follows the path of least time. [Snell's Law](#) can be derived from this by setting the [derivative](#) of the time =0. We make use of the [index of refraction](#), defined as  $n=c/v$ .



$$t = \frac{\sqrt{x^2 + a^2}}{c/n_1} + \frac{\sqrt{(d-x)^2 + b^2}}{c/n_2}$$

to minimize the time required we set the derivative of the time with respect to x equal to zero ( Fermat's principle)

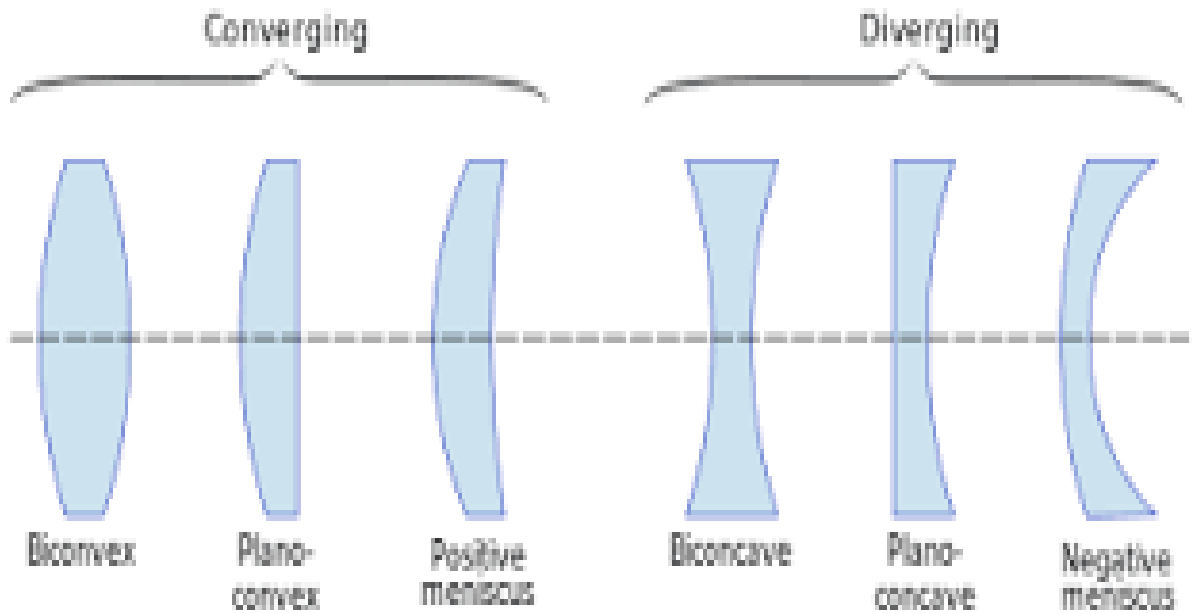
$$\frac{dt}{dx} = \frac{n_1 x}{c \sqrt{x^2 + a^2}} + \frac{-n_2 (d-x)}{c \sqrt{(d-x)^2 + b^2}} = 0$$

$$\frac{n_1 x}{\sqrt{x^2 + a^2}} = \frac{n_2 (d-x)}{\sqrt{(d-x)^2 + b^2}}$$

$n_1 \sin \theta_1 = n_2 \sin \theta_2$

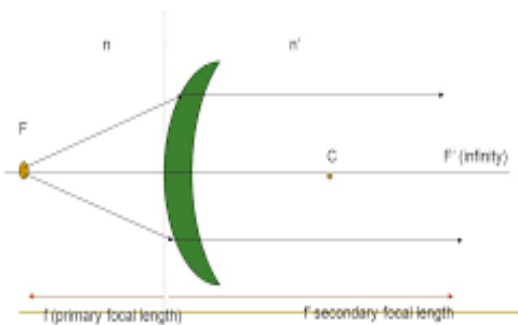
snell's law

**CH3-spherical surfaces**

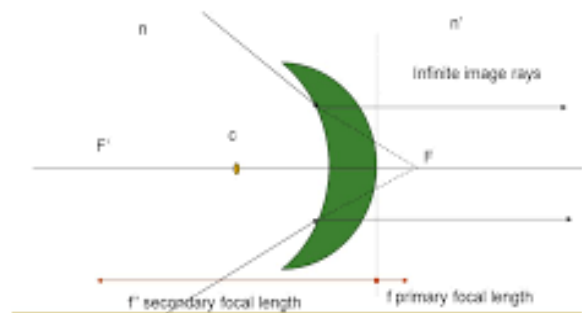


**focal points and focal lengths:**

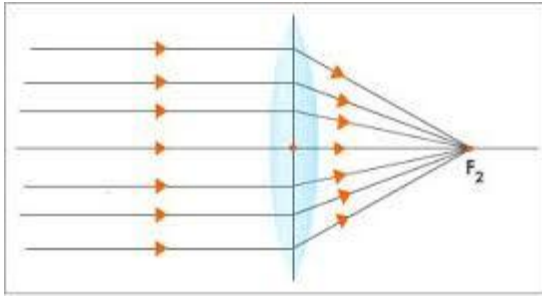
**Primary focal point- positive surface**



**Primary focal point – negative surface**

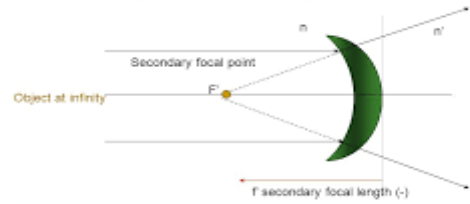


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### Secondary focal point –concave surface

Rays diverge as if they came from secondary focal point



principal axis: straight line pass through the center of curvature.

primary focal point( $F$ ): is an axial point having property that any ray coming from it or proceeding toward it travel parallel to the axis after refraction.

secondary focal point( $F'$ ): is an axial point having property that any incident ray travelling to the axis will after refraction proceed toward or appear to come from  $F'$ .

$$\frac{f'}{f} = \frac{n'}{n}$$

$f$ =primary focal length

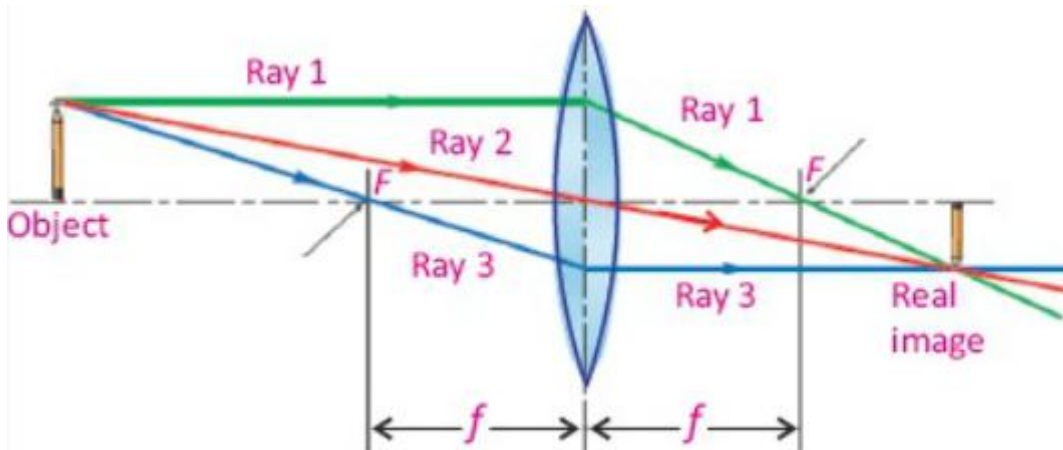
$f'$ =secondary focal length

$n, n'$  =indices of refraction

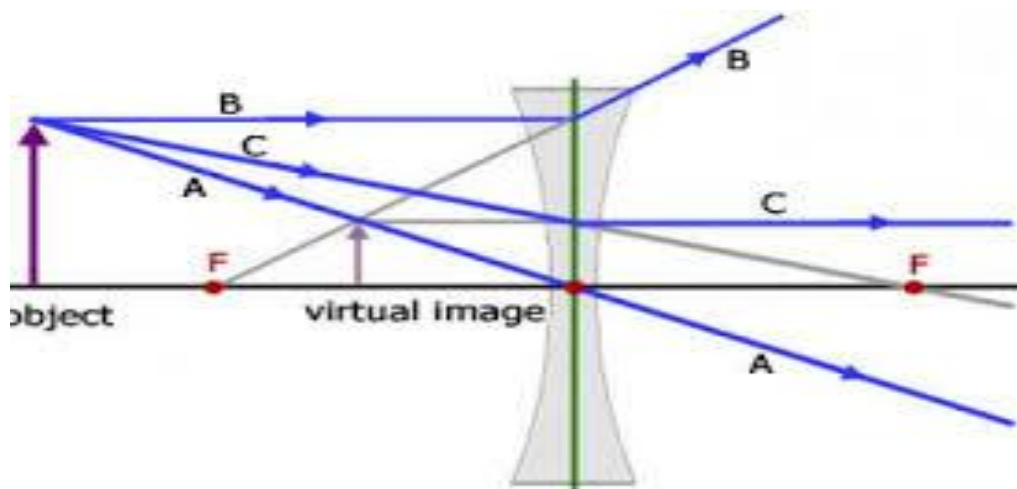
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### image formation

1- when the object is moved closer to the primary focal plane the image will be formed farther to the right away from ( $F$ ) and will be larger (magnified).



2-if the object is moved to the left, farther away from ( $F$ ) the image will be formed closer to ( $F$ ) and will be smaller in size.



## GEOMETRICAL OPTICS

### convention of signs

1-all figures are drawn with the light travelling from left to right.

2-all object distances( $S$ ) are considered as positive when they are measured to the left of the vertex and negative when they are measured to the right.

3-all image distances( $S'$ ) are positive when they are measured to the right of the vertex and negative when they are to left.

4-both focal lengths are positive for converging system and negative for diverging system.

5-object and image dimensions are positive when measured upward to the axis and negative when measured downward.

6-all convex surfaces encountered are taken as having a positive radius and all concave surfaces encountered are taken as a negative radius.

we can determine the position and size of the image by :

1-graphical method

2-experimental method

3-calculation using Gaussian formula.

for a single spherical surface:

$$\frac{n}{S} + \frac{n'}{S'} = \frac{n' - n}{r}$$

**Gaussian formula**

$n$ =index of refraction of the first medium

$n'$ = index of refraction of the second medium

$S$ =distance from object to the surface

$S'$ =distance from image to the surface

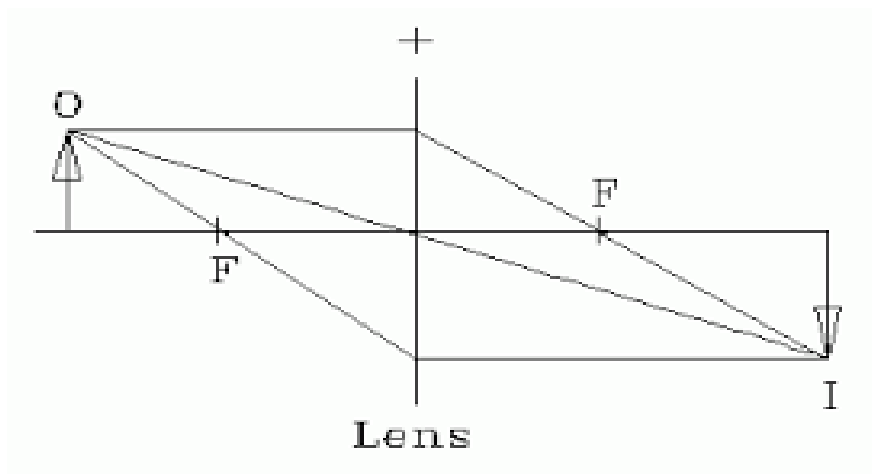
$r$  = radius of curvature.

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2-Graphical constructions:

a- parallel-ray method:

1- convex spherical surface: when  $n_2 > n_1$



2- concave spherical surface: when  $n_2 > n_1$

## GEOMETRICAL OPTICS

3-oblique-ray method:

### Magnification

It is the ratio between the transverse dimension of the final image and the corresponding dimension of the original object.

$$m = \frac{y}{y} = - \left( \frac{s-r}{s+r} \right)$$

note: if  $m$  is negative  $\Rightarrow$  the image is real and inverted

if  $m$  is positive  $\Rightarrow$  the image is virtual and erect

## GEOMETRICAL OPTICS

Reduce vergence:-

$$V = \frac{n}{s} \quad , \quad V = \frac{n}{s}$$

$$\frac{1}{S} + \frac{1}{S} = \frac{1}{f}$$

power of the surface

$$p = \frac{n}{f} \quad , \quad p = \frac{n}{f}$$

curvature of surface:

$$k = \frac{1}{r}$$

note: when all distances are measured in meters the reduced vergence  $V$ ,  $V$ , the curvature  $k$ , and the power ( $p$ ), are in units called (diopter) (D).

$$P = \frac{n-n}{r} = k (n - n)$$

$$V + V = P$$



## GEOMETRICAL OPTICS

EX: one end of a glass rod of refractive index 1.5 is ground and polished with a convex spherical surface of radius 10cm. An object is placed in the air on the axis 40cm to the left of the vertex, find:

- a- the power of the surface.
- b- the position of the image by Gaussian formula.
- c- the position of the image by reduce vergence.
- d- curvature of the surface.

sol.

$$a- p = \frac{n-n}{r} = \frac{1.5-1}{0.1} = 5 \text{ D}$$

$$b- \frac{n}{s} + \frac{n}{s} = \frac{n-n}{r} \quad \Rightarrow \quad \frac{1}{0.4} + \frac{1.5}{s} = \frac{1.5-1}{0.1}$$

$$s = 60 \text{ cm} = 0.6 \text{ m}$$

$$c- V = \frac{n}{s} = \frac{1}{0.6} = 1.67 \text{ D}$$

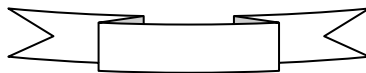
$$P = V + V$$

$$5 = 2.5 + V \quad \Rightarrow \quad V = 2.5 \text{ D}$$

$$V = \frac{n}{s}$$

$$2.5 = \frac{1.5}{s} \quad \Rightarrow \quad s = 0.6 \text{ m}$$

$$d- k = \frac{1}{r} = \frac{1}{0.1} = 10 \text{ D}$$



EX: The left end of glass rod of index (1.6) is ground and polished to a convex spherical surface of radius 3cm. A small object is located in the air on the axis 10cm from the vertex, find:

- a- the primary and secondary focal lengths.
- b- the surface power.
- c- the image distance.
- d- the lateral magnification.

sol.

## GEOMETRICAL OPTICS

$$a- \frac{n}{f} = \frac{n-n}{r} \Rightarrow \frac{1}{f} = \frac{1.6-1}{3} \Rightarrow f = 5\text{cm}$$

$$\frac{n}{f} = \frac{n-n}{r} \Rightarrow \frac{1.6}{f} = \frac{1.6-1}{3} \Rightarrow f = 8\text{ cm}$$

$$b- P = \frac{n}{f} = \frac{1}{0.05} = 20\text{D}$$

$$c- \frac{n}{s} + \frac{n}{s} = \frac{n-n}{r} \Rightarrow \frac{1}{10} + \frac{1.6}{s} = \frac{1.6-1}{3} \Rightarrow s = 16\text{cm}$$

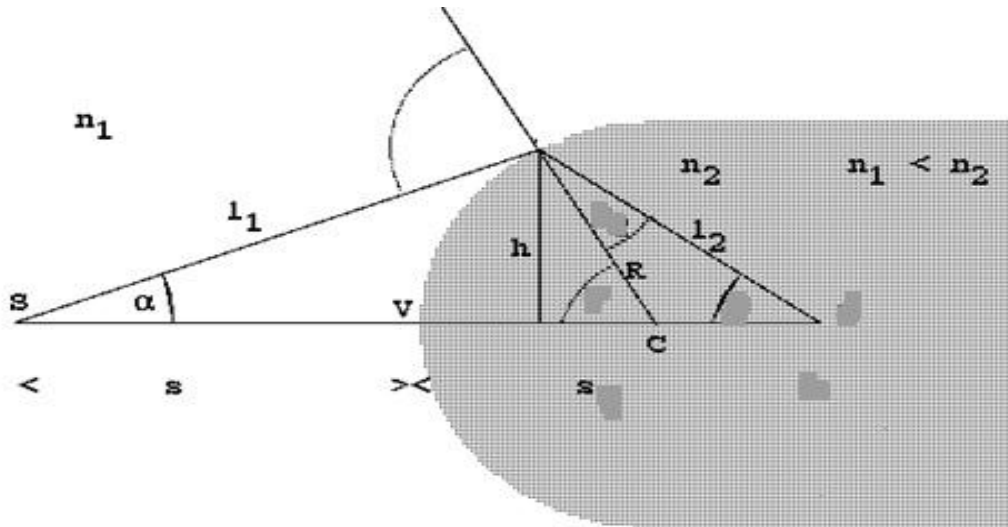
$$d- m = -\left(\frac{s-r}{s+r}\right) = -\left(\frac{16-3}{16+3}\right) = -1$$

m is negative  $\Rightarrow$  the image is real and inverted



## GEOMETRICAL OPTICS

### Derivation of the Gaussian formula



This figure shows the refraction of ray of light at the convex surface to form a real image for the object .

in the figure (TA) is the spherical surface separates between two media.

the first medium: refractive index( $n$ ) ,contains the object at point M.

the second medium: refractive index( $n'$ ) ,centre of curvature at point c.

the incident ray MT incident on the convex surface at T with refraction angle  $\phi$  in the medium of refractive index ( $n$ ),this ray refracts in the second medium of refractive index  $n'$  with refraction angle( $\phi'$  ).the intersection of the refracted ray with the original axis at  $M'$  (image position).

M- object point

$\phi$ - incident angle

$\phi'$  -refractive angle

when the incident and refracted rays MT,  $TM'$  are paraxial.

the angles  $\phi$ ,  $\phi'$  will be small so,we put the sines of the angles equal to the angles themselves and :

from snell's law:

$$n \sin \phi = n' \sin \phi'$$

## GEOMETRICAL OPTICS

$$n \phi = n' \phi' \implies \frac{\phi}{\phi'} = \frac{n}{n'} \text{ -----(1)}$$

from the geometry of the fig.:-  $\Delta MTC$

$$\phi = \alpha + \beta \text{ -----(2)}$$

$\phi$  -exterior angle

$\alpha, \beta$  - are interior angles

for  $\Delta TCM'$  :

$$\beta = \phi' + \gamma$$

$\beta$  -is an exterior angle

$\phi', \gamma$  - are interior angles

$$\phi' = \beta - \gamma \text{ -----(3)}$$

substitute  $\phi, \phi'$  in (1):-

$$\frac{\alpha + \beta}{\beta - \gamma} = \frac{n}{n'}$$

$$n' \beta - n' \gamma = n \alpha + n \beta$$

$$n \alpha + n' \gamma = (n' - n) \beta \text{ -----(4)}$$

$\alpha, \beta, \gamma$  are very small angles

$$\alpha = \frac{h}{s}, \beta = \frac{h}{r}, \gamma = \frac{h}{s'} \text{ -----(5)}$$

substitute (5) in (4) :-

$$n \frac{h}{s} + n' \frac{h}{s'} = (n' - n) \frac{h}{r}$$

by cancelling (h):-

$$\frac{n}{s} + \frac{n'}{s'} = \frac{(n' - n)}{r}$$

Gaussian formula

this relation connects between the object and it's image which is formed by refraction at convex surface.

$$\frac{(n' - n)}{r} = \text{power of the surface}$$

= positive if the surface converge the incident rays.

= negative if the surface diverge the incident rays

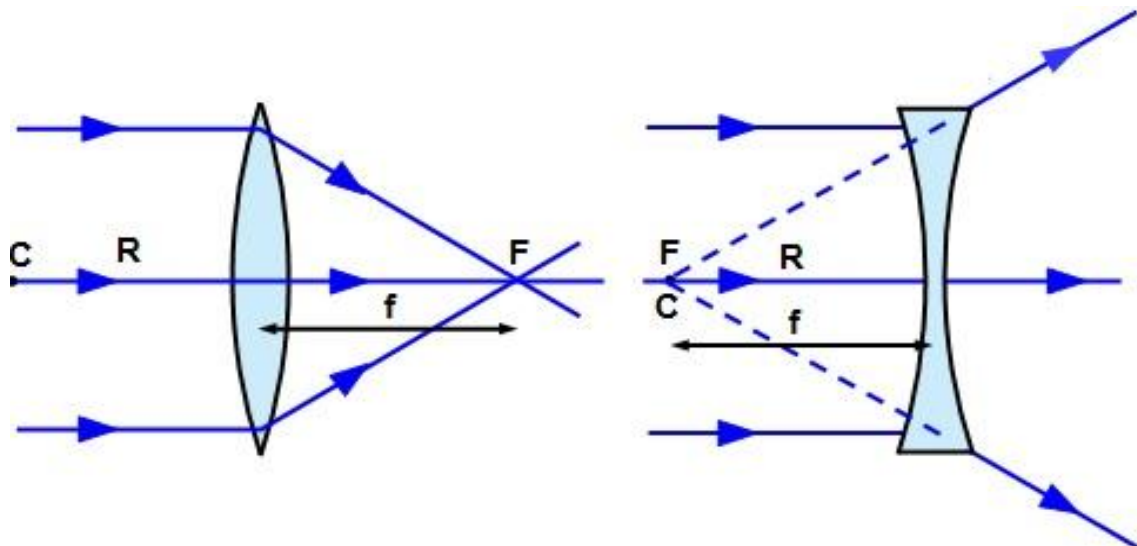
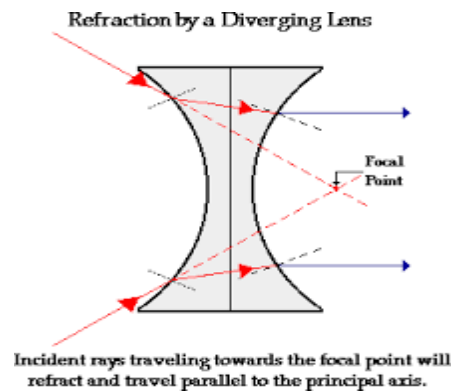
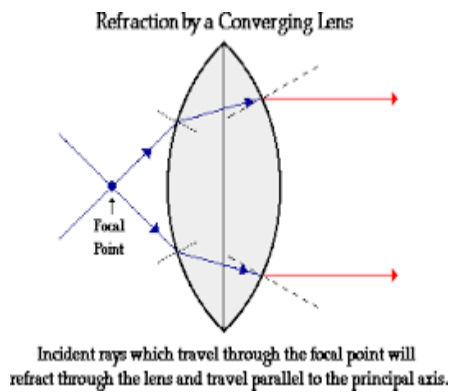
# GEOMETRICAL OPTICS

## CH-4

### THIN LENSES

**THIN LENS**: A thin lens may be defined as one whose thickness is considered small in comparison with the distance generally associated with optical properties.

such distances are for example radii of curvature of two spherical surfaces ,primary and secondary focal length and object and image distances.



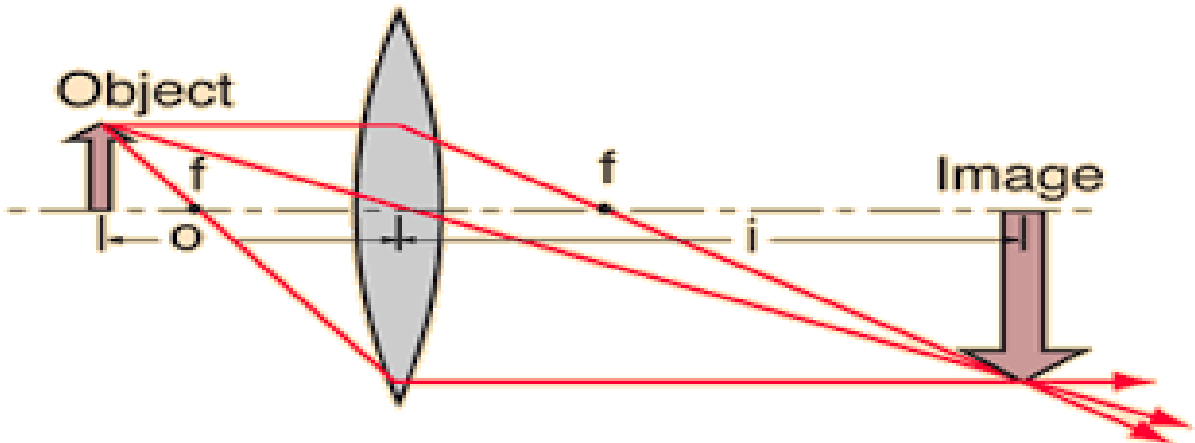
## GEOMETRICAL OPTICS

**The primary focal point( $F$ ):** is an axial point having the property that any ray coming from it or proceeding toward it, travels parallel to the axis after refraction.

**secondary focal point( $F'$ ):** is an axial point having the property that any incident ray travelling parallel to the axis will after refraction proceed toward or appear to come from  $F'$ .

$F, F'$  are measured in centimeter or inches, having a positive sign for converging lenses, and negative sign for diverging lenses.

### Image formation



when an object is placed on one side or the other of a converging lens and beyond the focal plane an image is formed on the opposite side.

1-if the object is moved closer to the primary focal plane, the image will be formed farther away from the secondary focal plane and will be larger (magnified).

2-when the object moved farther away from ( $F$ ),the image will be formed closer to ( $F'$ )and will be smaller in size.

### conjugate points and planes

any points of object and image points such as  $M$  and  $M'$  are called conjugate points, and planes passes through these points perpendicular to the axis are called conjugate planes.

## GEOMETRICAL OPTICS

### Determination the position of the image

There are three methods to determine the position of the image:-

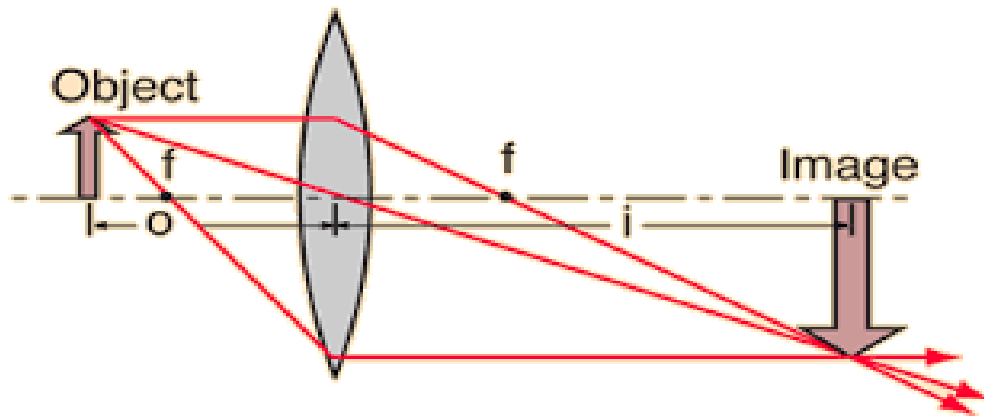
1- graphical construction

2- experimental method

3-the lens formula ( $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ )

1- graphical construction

a- the parallel-ray method



b- the oblique-ray method

## GEOMETRICAL OPTICS

3- use of the lens formula

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \Rightarrow \quad s' = \frac{s \cdot f}{s - f}$$

### convention of signs

the sign convention of thin lens are identical to those for a single spherical surface.

### lateral magnification

from fig. b  $y = MQ, y' = M'Q'$

$$m = \frac{y'}{y}$$

when :  $m$  is negative  $\Rightarrow$  inverted image

$m$  is positive  $\Rightarrow$  erect image

note: in converging lens the virtual image will be formed when the image locates between the primary focal point and the lens.



## GEOMETRICAL OPTICS

### LENS MAKERS FORMULA

A lens is to be ground to some spherical focal length, refractive index of the glass must be known.

supporting the index to be chosen so as to satisfy the equation:

$$\frac{1}{f} = (n - 1) \left( \frac{1}{r} - \frac{1}{r} \right)$$

substituting  $\left( \frac{1}{s} + \frac{1}{s} = \frac{1}{f} \right)$

$$\frac{1}{s} + \frac{1}{s} = (n - 1) \left( \frac{1}{r} - \frac{1}{r} \right)$$

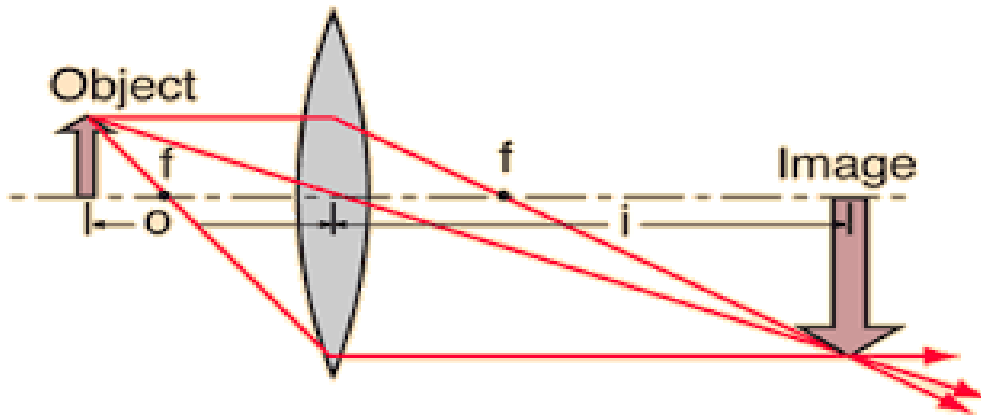
$r_1$  = positive for convex lens

$r_2$  = negative for concave lens

## GEOMETRICAL OPTICS

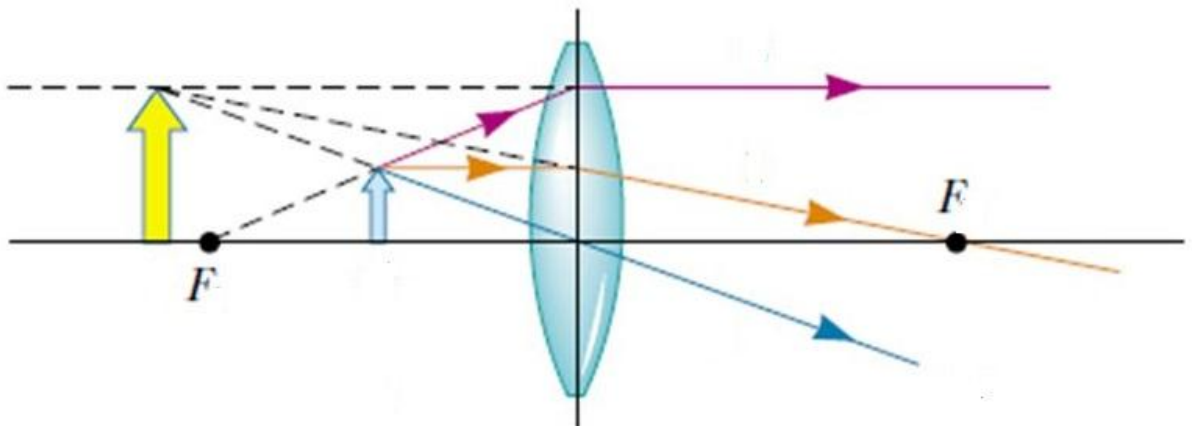
### virtual image

1- the image formed by the converging lens is real and can be made visible on a screen.



real image formed by converging lens

A virtual image cannot be formed on a screen, **the virtual image with converging lens may be formed just in the case when the object locates between the primary focal point and the lens.**

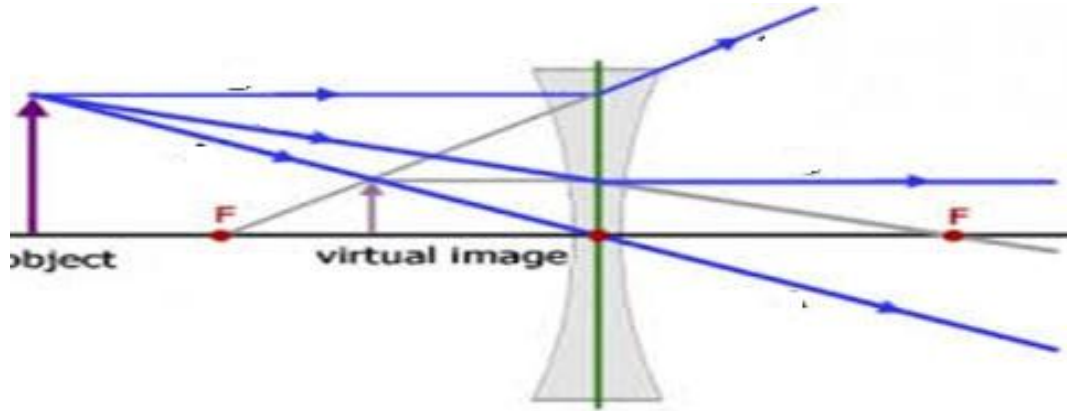


virtual image by converging lens

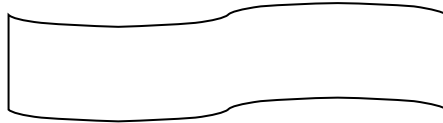
## GEOMETRICAL OPTICS

### 2- With diverging lens, the image will be always virtual

i.e cannot be viewed on a screen since rays are diverging on the right of the lens.



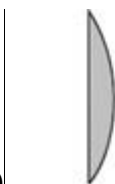
virtual image by diverging lens



Ex: A plano-concave lens having a focal length of 25cm is to be made of glass of index ( $n=1.52$ ). Calculate the radii of curvature of the grinding and polishing tools that must be used to make this lens.

sol:

$$r_2 = \infty$$

$$\frac{1}{f} = (n-1) \left( \frac{1}{r} - \frac{1}{r} \right)$$


$$\frac{1}{25} = (1.52 - 1) \left( \frac{1}{r} - \frac{1}{\infty} \right)$$

$$r_1 = 13 \text{ cm}$$

## GEOMETRICAL OPTICS

### The power of thin lens

$$P = \frac{1}{f}$$

is the reciprocal of the focal length in meters, the power of thin lens measured with Diopters .

$$\text{Diopter} = \frac{1}{m}$$

converging lens have positive power

diverging lens have negative power

$$p = \frac{1}{f} = (n-1) \left( \frac{1}{r} - \frac{1}{r} \right)$$

Ex: The radii of both surfaces of an equiconvex lens of  $n= 1.6, r_1=0.08m,$   
 $r_2=- 0.08m,$  find the lens power.

$$p = (n-1) \left( \frac{1}{r} - \frac{1}{r} \right)$$

$$p = (1.6 - 1) \left( \frac{1}{0.08} - \frac{1}{-0.08} \right)$$

$$p = 0.6 \left( \frac{1}{0.08} + \frac{1}{0.08} \right)$$

$$p = 0.6 * 25 = 15D$$



## GEOMETRICAL OPTICS

### Thin lens combination

1- apply imaging formula to the first lens to find image for this lens

$$s_1' = \frac{s f}{s - f}$$

2- find object distance for the second lens(negative sign means virtual object)

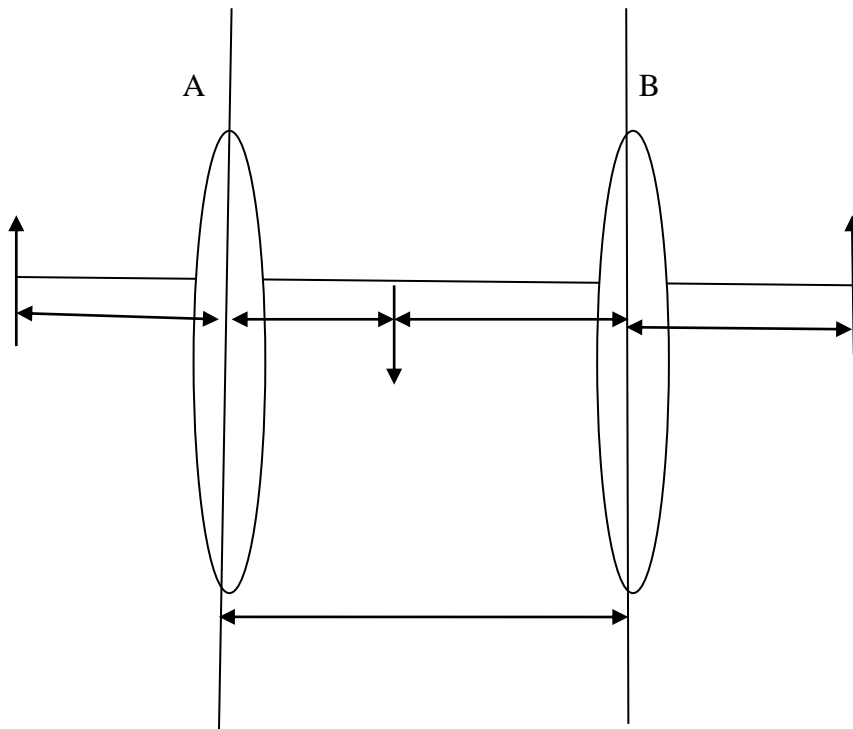
$$s_2 = d - s_1'$$

3- use imaging formula again to find the final image.

$$s_2' = \frac{s f}{s - f}$$

Ex: two converging lenses, A and B, with focal lengths  $f_A = 20\text{cm}$ ,  $f_B = 25\text{cm}$ , are placed  $80\text{cm}$  apart. An object is placed  $60\text{cm}$  in front of the first lens.

determine : a- position, and b- the magnification of the final image formed by the combination of the two lenses.



## GEOMETRICAL OPTICS

$$S_1^- = \frac{s f}{s - f} = \frac{60 \cdot 20}{60 - 20} = \frac{1200}{40} = 30 \text{ cm}$$

$$s_2 = 50 \text{ cm}$$

$$S_2^- = \frac{s f}{s - f} = \frac{50 \cdot 25}{50 - 25} = \frac{1250}{25} = 50 \text{ cm}$$

$$m_2 = -\frac{50}{50} = -1$$

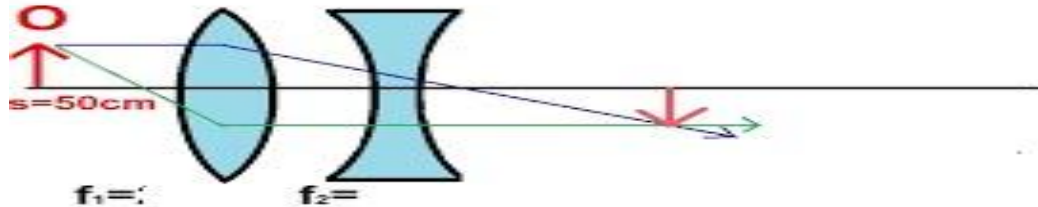
$$m = m_1 \cdot m_2$$

$$m = -0.5 \cdot -1 = +0.5$$

the image is the half the size of the object and upright.

## GEOMETRICAL OPTICS

Ex: two lenses, a converging lens A and a diverging lens B with  $f_A = 30\text{cm}$ ,  $f_B = 50\text{cm}$ , are placed  $50\text{cm}$  apart. An object is placed  $50\text{cm}$  in front of the first lens. determine the position and the magnification of the final image formed by the combination of the two lenses.



$$s_1' = \frac{s f}{s - f} = \frac{50 \cdot 30}{50 - 30} = \frac{1500}{20} = 75\text{cm}$$

$$s_2 = 50\text{cm}$$

$$s_2' = \frac{s f}{s - f} = \frac{-25 \cdot 50}{-25 - 50} = \frac{-1250}{-75} = -16.67\text{cm}$$

magnification:

$$m_1 = -\frac{75}{50} = -1.5$$

$$m_2 = \frac{-16.67}{-25} = 0.667$$

$$m = m_1 \cdot m_2$$

$$m = -1.5 \cdot 0.667 = -1.0$$

# GEOMETRICAL OPTICS

## CH-5

### THICK LENSES

1- we find the primary focal length for the first surface

$$\frac{n}{f} = \frac{n-n}{r} \quad \longrightarrow \quad f_1 = ?$$

2-- we find the secondary focal length for the first surface

$$\frac{n}{f} = \frac{n-n}{r} \quad \longrightarrow \quad f_1' = ?$$

3-- we find the primary focal length for the second surface

$$\frac{n}{f} = \frac{n-n}{r} \quad \longrightarrow \quad f_2' = ?$$

4-- we find the secondary focal length for the second surface

$$\frac{n}{f} = \frac{n-n}{r} \quad \longrightarrow \quad f_2 = ?$$

5-- we find the primary and secondary focal lengths for the system

$$\frac{n}{f} = \frac{n}{f} + \frac{n}{f} - \frac{d n}{f f}$$

$\longrightarrow$  primary focal length for the system  $f = ?$

$$\frac{n}{f} = \frac{n}{f}$$

$$f_2' = ?$$

6- we find the and secondary focal points for the system

$$A_1F = -f \left( 1 - \frac{d}{f} \right) \text{ primary focal point for the system}$$

$$A_2F = +f \left( 1 - \frac{d}{f} \right) \text{ secondary focal point for the system}$$



## GEOMETRICAL OPTICS

7- we find the primary principal point and secondary principal point

$$A_1H = + f \frac{d}{f}$$

$$A_2H = - f \frac{d}{f}$$

---

For the purpose of confirming the accuracy of the results:

a-  $A_1F - A_1H = FH = f$  (primary focal length for the system)

b-  $A_2F - A_2H = FH = f_2$  (secondary focal length for the system)

---

**Ex:** A lens has the following specifications:  $r_1 = + 1.5$  cm,  $r_2 = 1.5$  cm,  $d = 2$  cm,  $n = 1$ ,  $n' = 1.6$ , and  $n'' = 1.3$ .

find: a- the primary and secondary focal lengths of the separate surfaces.

b- the primary and secondary focal lengths of the system. and c- the primary and secondary principal point

solution:

لكي نجد البعد البؤري الاولى للسطح الاول:

$$a- \frac{n}{f} = \frac{n-n}{r} = \frac{1.6-1}{1.5} = 0.4$$

$$\frac{1}{f} = 0.4 \quad \Longrightarrow \quad f_1 = 2.5 \text{ cm}$$

لكي نجد البعد البؤري الثانوى للسطح الاول:

$$\frac{n}{f} = \frac{n-n}{r} = \frac{1.6-1}{1.5} = 0.4$$

$$\frac{1.6}{f} = 0.4 \quad \Longrightarrow \quad f_1' = 4 \text{ cm}$$

## GEOMETRICAL OPTICS

لكي نجد البعد البؤري الاولى للسطح الثاني:

$$\frac{n}{f} = \frac{n-n}{r} = \frac{1.3-1.6}{1.5} = -0.2$$

$$\frac{1.6}{f} = -0.2 \implies f_2 = -8 \text{ cm}$$

لكي نجد البعد البؤري الثاني للسطح الثاني:

$$\frac{n}{f} = \frac{n-n}{r} = -0.2$$

$$\frac{1.3}{f} = -0.2 \implies f_2 = -6.5 \text{ cm}$$

حساب الابعاد البؤرية الاولى والثانية للنظام:

$$\frac{n}{f} = \frac{n}{f} + \frac{n}{f} - \frac{dn}{ff}$$

$$\frac{n}{f} = \frac{1.6}{4} + \frac{1.3}{-6.5} - \frac{2*1.3}{4*(-6.5)}$$

$$\frac{n}{f} = 0.4 - 0.2 + 0.1 = 0.3$$

$$\frac{1}{f} = 0.3 \implies f = \frac{1}{0.3} = 3.333 \text{ cm} \quad \text{البعد البؤري الاولى للنظام}$$

## GEOMETRICAL OPTICS

$$\frac{n}{f} = \frac{n}{f} = 0.3$$

$$\frac{n}{f} = 0.3 \quad \Rightarrow \quad \frac{1.3}{f} = 0.3$$

$$f_2 = \frac{1.3}{0.3} = 4.333 \text{ cm} \quad \underline{\text{البعد البؤري الثانوي للنظام}}$$

لحساب النقاط البؤرية الاولية والثانوية للنظام:

$$A_1 F = -f \left( 1 - \frac{d}{f} \right) = -3.33 \left( 1 - \frac{2}{-8} \right) = -4.166 \text{ cm} \quad \underline{\text{النقطة البؤرية الاولية للنظام}}$$

$$A_2 F = -f \left( 1 - \frac{d}{f} \right) = +4.333 \left( 1 - \frac{2}{4} \right) = 2.167 \text{ cm} \quad \underline{\text{النقطة البؤرية الثانوية للنظام}}$$

### problem 1(ch-5):

An equiconvex lens has an index of 1.8, radii of 4 cm and a thickness of 3.6 cm.  
calculate : a- the focal length. b- the distances from the vertices to the corresponding focal points and principal points.

$$n = 1.8, n = 1, n = 1.8, d = 3.6 \text{ cm}, r_1 = r_2 = 4 \text{ cm}$$

$$\frac{n}{f} = \frac{n}{f} = \frac{n - n}{r}$$

$$\frac{1}{f} = \frac{1.8 - 1}{4} \quad \Rightarrow \quad f_1 = 5 \text{ cm}$$

$$\frac{n}{f} = \frac{1.8 - 1}{4} \quad \Rightarrow \quad \frac{1.8}{f} = \frac{1.8 - 1}{4}$$

## GEOMETRICAL OPTICS

$$f_1' = 9 \text{ cm}$$

$$\frac{n}{f} = \frac{n}{f} = \frac{n - n}{r}$$

$$\frac{n}{f} = \frac{n - n}{r} \Rightarrow \frac{1.8}{f} = \frac{1 - 1.8}{4}$$

$$\Rightarrow f_2 = -9 \text{ cm}$$

$$\frac{n}{f} = \frac{n - n}{r} = \frac{1}{f} = \frac{1 - 1.8}{4}$$

$$\Rightarrow f_2 = -5 \text{ cm}$$

b-  $p = \frac{n}{f}$

$$\frac{n}{f} = \frac{n}{f} = \frac{n}{f} + \frac{n}{f} - \frac{dn}{ff}$$

$$\frac{1}{f} = \frac{1.8}{9f} + \frac{1}{-5} - \frac{3.6 * 1}{-5 * 9} = 0.2 - 0.2 + 0.08$$

$$f = 12.5 \text{ cm} = f$$

c-

حساب النقاط الاساسية الاولى والثانوية

$$A_1H = +f \frac{d}{f} = +3.33 \frac{2}{-8} = -0.833 \text{ cm}$$

$$A_2H = -f \frac{d}{f} = -4.33 \frac{2}{4} = -2.167 \text{ cm}$$

## GEOMETRICAL OPTICS

وكتحقيق لقيمة البعد البؤري الاولى للنظام:

$$A_1F - A_1H = -4.166 - (-0.833)$$

$$F H = -3.333\text{cm}$$

البعد البؤري الاولى للنظام

$$A_2F - A_2H = 2.167 - (-2.165)$$

$$F H = 4.334\text{ cm}$$

البعد البؤري الثانوي للنظام

# GEOMETRICAL OPTICS

## CH-6

### SPHERICAL MIRRORS

A spherical reflecting surface has image forming properties similar to those of a thin lens or of a single reflecting surface. The image from a spherical mirror is in some respects superior to that from a lens, notably in the absence of chromatic white light. Therefore mirrors are occasionally used in place not so broad as those of lenses because they do not offer aberrations of the image.

Because of the simplicity of the law of reflection as compared to the law of refraction, the quantitative study of image formation by mirrors is easier than in the case of lenses. Many features are the same of lenses, and there are some characteristics which are different.

#### Focal point and focal length

Diagrams showing the reflection of a parallel beam of light by a convex mirror and by a concave mirror .

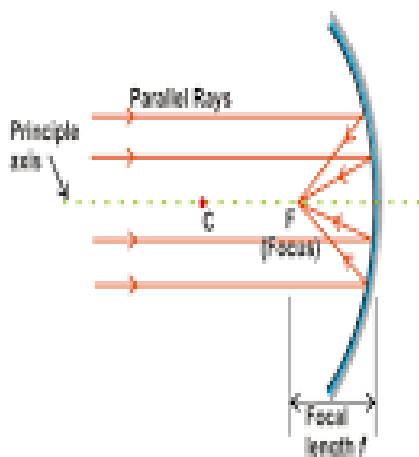


Figure 3a. Concave Mirror

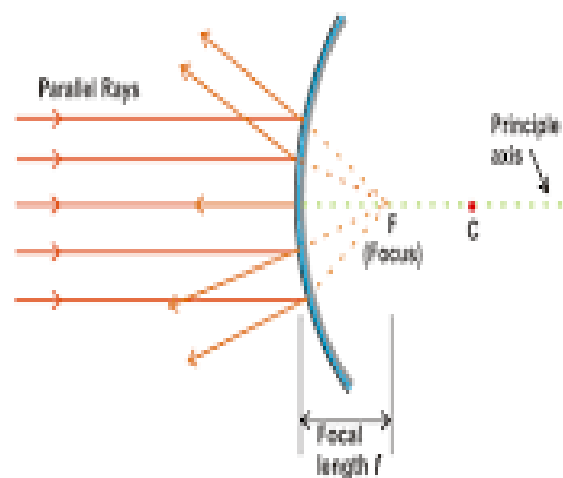


Figure 3b. Convex Mirror

## GEOMETRICAL OPTICS

A ray striking the mirror at some point such as T obeys the law of reflection  $\phi = \phi$ . All rays are shown as brought to a common focus at F, although this will be strictly true for paraxial rays. The point F is called the focal point and the distance FA the focal length.

In the second diagram the reflected rays diverge as though they come from a common point F. since the angle TCA also equal  $\phi$ , the triangle TCF is an isosceles one, and in general  $CF=FT$ , but for very small angles  $\phi$  (paraxial rays),  $FT$  approaches equality with  $FA$ , hence

$$FA = \frac{1}{2} (CA) \text{ or } f = -\frac{1}{2} r$$

and for focal length equals one-half the radius of curvature.

**for concave mirror : focal length  $\Rightarrow$  is positive**

**radius of curvature  $\Rightarrow$  is positive**

**for convex mirror: focal length  $\Rightarrow$  is negative**

**radius of curvature  $\Rightarrow$  is negative**

and for mirror there is one focal point.

object distance  $S$  and image distance are measured for the object and for image respectively to the vertex. this make both  $S$  and  $S'$  positive and the object and image real when they lie to the left of the vertex, while they are negative and virtual when they lie to the right.

## GEOMETRICAL OPTICS

### DERIVATION OF MIRROR FORMULA

by the law of reflection the radius CT bisects the angle  $\angle M T M'$ , using geometrical theorem we may write the proportion:

$$\frac{MC}{MT} = \frac{C M'}{M T'} \quad \text{-----(1)}$$

Now, for paraxial ray,  $MT = MA = S$ , and  $M T' = M A' = S'$ ,

$$MC = MA - CA = s - (-r) = s + r \quad \text{-----(2)}$$

$$CM' = CA - M A' = -r - s = -(s + r) \quad \text{-----(3)}$$

by substituting eq.(2), and eq.(3) in eq.(1):

$$\frac{s+r}{s} = - \frac{s+r}{s}$$
$$\frac{s}{s} + \frac{r}{s} = - \frac{s}{s} - \frac{r}{s}$$

$$1 + \frac{r}{s} = -1 - \frac{r}{s}$$



## GEOMETRICAL OPTICS

$$\frac{r}{s} + \frac{r}{s} = -2 \implies r \left( \frac{1}{s} + \frac{1}{s} \right) = -2$$

$$\boxed{\frac{1}{s} + \frac{1}{s} = \frac{-2}{r}} \quad \text{-----(4) mirror formula}$$

The primary focal point is defined as that axial point object point for which the image is formed at infinity, so substituting  $s = f$ ,  $s = \infty$ , in eq.(4), we have:

$$\frac{1}{f} + \frac{1}{\infty} = \frac{-2}{r}$$

from which  $\frac{1}{f} = \frac{-2}{r}$  or  $f = -\frac{r}{2}$  -----(5)

The secondary focal point is defined as the image point of an infinite distant object point. this is,  $s = f$ ,  $s = \infty$ , so that:

$$\frac{1}{\infty} + \frac{1}{f} = \frac{-2}{r}$$

$$\frac{1}{f} = \frac{-2}{r} \implies f = -\frac{r}{2} \quad \text{-----(6)}$$

## GEOMETRICAL OPTICS

$$\frac{1}{f} = \frac{1}{f} = \frac{-2}{r} \quad \text{-----}(7)$$

substitute eq.(7) in eq.(4) ,we have:

$$\boxed{\frac{1}{s} + \frac{1}{s} = \frac{1}{f}} \quad \text{-----}(8) \quad \text{just as for lenses.}$$

from the proportionality of sides in the similar triangles Q A M ,and QAM in fig.

( ) ,we find  $\frac{-y}{y} = \frac{s}{s}$  giving:

$$\mathbf{m} = \frac{y}{y} = - \frac{s}{s}$$

---

## GEOMETRICAL OPTICS

EX: An object is located 20cm in front of a convex mirror of radius 50cm. calculate (a) the power of the mirror.(b) the position of the image,(c) it's magnification.

sol.

$$a- k = \frac{1}{r} = \frac{1}{0.5} = + 2 \text{ D} \text{ and } V = \frac{1}{s} = \frac{1}{0.2} = + 5 \text{ D}$$

$$P = - 2 k = - 2 * 2 = - 4 \text{ D}$$

$$V + V = P$$

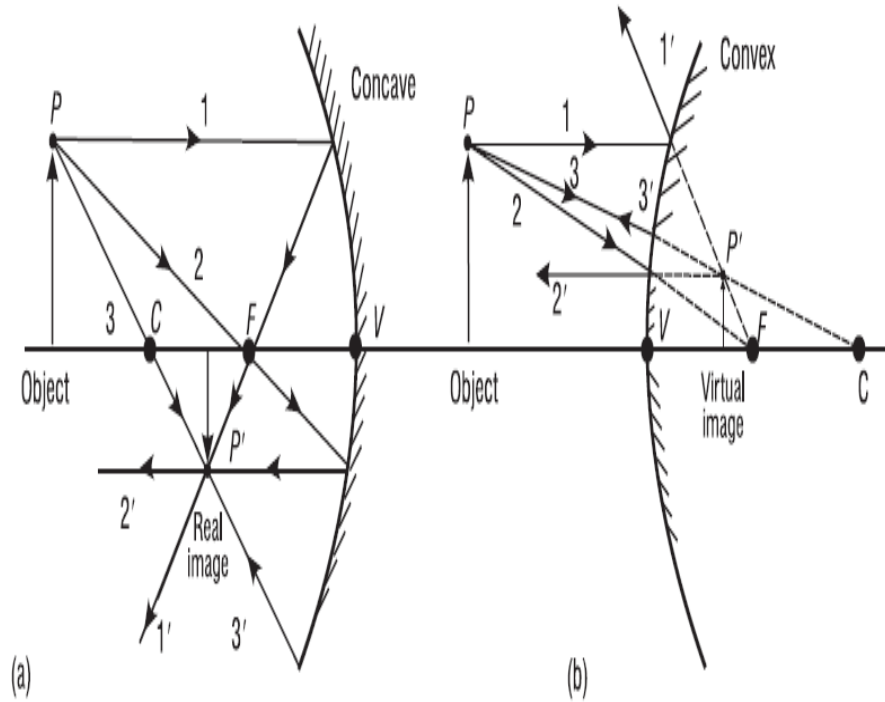
$$5 + V = - 4 \text{ or } V = -9 \text{ D}$$

$$S = \frac{1}{v} = - \frac{1}{9} = - 0.111 \text{ m} = -11.1 \text{ cm}$$

$$m = - \frac{s}{s'} = - \frac{v}{u} = - \frac{5}{-9} = 0.555$$

the image is virtual and erect located at 11.1cm to the right of the mirror and has magnification 0.555x.

# GEOMETRICAL OPTICS



**concave mirror**

**convex mirror**

**image formation by convex and concave mirror**